hands-on mathematics

Grade 3

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Introduction to *Hands-On Mathematics*

Program Introduction

Hands-On Mathematics focuses on developing students' knowledge, skills, and attitudes through active inquiry, problem solving, and decision making. Throughout all activities, students are encouraged to explore, investigate, and ask questions in order to heighten their own curiosity about and understanding of the world of mathematics.

Program Principles

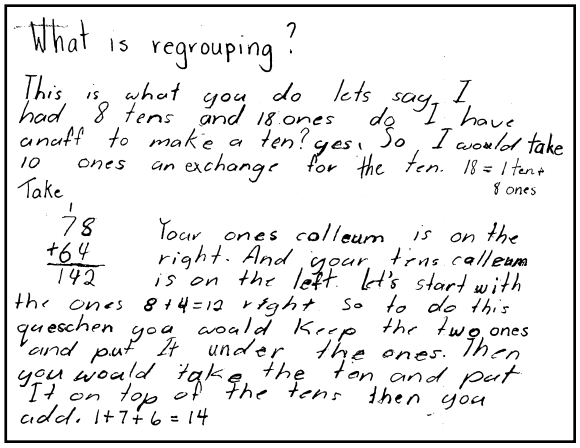
1. Effective mathematics programs involve students actively building new knowledge from experience and prior knowledge.
2. The development of students' understanding of concepts, flexibility in thinking, reasoning, and problem-solving skills/strategies form the foundation of the mathematics program.
3. From a young age, children are interested in mathematical ideas. This interest must be maintained, fostered, and enhanced through active learning.
4. Mathematics activities must be meaningful, worthwhile, and relate to real-life experiences.
5. The teacher's role in mathematics education is to actively engage students in tasks and experiences designed to deepen and connect their knowledge. Children learn best by doing, rather than by just listening. The teacher, therefore, should focus on creating opportunities for students to interact, in order to propose mathematical ideas and conjectures, to evaluate their own thinking and that of others, and to develop mathematical reasoning skills.
6. Mathematics should be taught in correlation with other school subjects. Themes and topics of study should integrate ideas and skills whenever possible.
7. The mathematics program should encompass, and draw on, a range of educational resources, including literature and technology, as well as people and places in the local community.
8. Assessment of student learning in mathematics should be designed to focus on performance and understanding, and should be conducted through meaningful and varied assessment techniques carried on throughout the units of study.

The Big Ideas of Mathematics

In order to achieve the goals of mathematics education and to support lifelong learning in mathematics, students must be provided with opportunities to encounter and practice critical mathematical processes. These processes are as follows:

Communication

Students need to be given opportunities to communicate their mathematical ideas through the use of oral language, reading and writing, diagrams, charts, tables, and graphs.
For example:



Program Implementation

Program Resources

Hands-On Mathematics is arranged in a format that makes it easy for teachers to plan and implement.

Units comprise the selected topics of study for the grade level, organized into lessons. The units relate directly to the learning expectations identified on pages 5 through 11, which are established by the *Ontario Curriculum for Mathematics (2005)*.

The introduction to each unit summarizes the general goals for the unit and provides background information for teachers. Each unit begins with a list of books for students that relate to the unit; a list of related websites (for all units combined) can also be found on pages 120 through 122.

Units are organized into lessons, based on the expectations.

Note: This does not imply that a lesson can be covered in only one lesson period; many will carry over several lesson periods.

The lessons are arranged in the following format:

Background Information for Teachers: Some lessons provide teachers with the basic mathematical knowledge they will need to present the activities. This information is offered in a clear, concise format, and focuses specifically on the topic of study.

Materials: A complete list of materials required to conduct the main activity or activities is provided. It includes classroom materials, equipment, and visuals. The quantity of materials required will depend on how you conduct activities and whether students are working individually or in groups.

Activity/Activities: This section details a step-by-step procedure, including higher-level questioning techniques and suggestions for encouraging active inquiry and discussion.

Activity Sheets: Reproducible activity sheets have been designed to correlate with the specific expectations of the activity or activities. Many of these are used during the activity to record results of investigations. Others are used as follow-up to the in-class activities. Students may work independently on these sheets, in small groups, or you may choose to read through them together and complete them in a large group setting. Activity sheets can also be made into overheads or large experience charts. Since it is also important for students to learn to construct their own charts and recording formats, these activity sheets can be used by the teacher as examples of ways to record and communicate ideas about an activity. Students can then create their own sheets rather than use the ones provided.

Note: Activity sheets are meant to be used only in conjunction with, or as a follow-up to, the hands-on activities. The activity sheets are not intended to be the mathematics lesson in itself or the sole assessment for the lesson.

Problem Solving: Many lessons include suggestions for problem-solving activities that are directly related to the lesson's expectations. These problems may be presented orally, acted out with concrete objects, presented pictorially, or written out on chart paper. At the end of each unit, many of these problems are presented again on black line masters. Teachers can copy these sheets onto overhead transparencies to present to students as daily problem-solving activities. Or, the masters can be copied for students and cut apart, problem by problem. Students can then paste the problems into their math journals or agendas for completion independently.

Activity Centre: Some lessons include independent student activities that focus on the expectations.

Extension: Many lessons include optional activities to extend, enrich, and reinforce the expectations.

Assessment Suggestions: Throughout each unit, several suggestions are made for assessing student learning. Again, these assessment strategies focus specifically on the learning expectations of a particular activity topic. In the next section of the ***Hands-On Mathematics*** program, assessment is dealt with in detail. Keep in mind that the suggestions made within activities are merely ideas to consider; you may use your own assessment techniques or refer to the other assessment strategies on pages 17 and 18.

Classroom Environment

The classroom setting is an important component of the learning process. An active environment – one that gently hums with the purposeful conversations and activities of students – indicates that meaningful learning is taking place. While studying a specific topic, the room should display related objects and materials, student work, pictures and posters, maps, graphs, and charts made during activities, and summary charts of important concepts taught and learned. These reinforce concepts and skills that have been stressed during mathematics activities.

Timelines

No two groups of students will cover topics and material at the same rate. Planning the duration of units is the responsibility of the teacher. In some cases, the activities described will not be completed during one block of time and will have to be carried over. (Division of units into "lessons" does not imply that they can be covered in only one lesson period. Many lessons will, in fact, carry over several lesson periods.) In other cases, you may observe that the students are especially interested in one topic, and you may choose to expand upon it. The individual needs of your students should be considered as there are no strict timelines involved in the ***Hands-On Mathematics*** program. It is important, however, to spend time on every unit in the program so that students focus on all of the expectations established for their grade level.

Classroom Management

Although active learning is emphasized throughout this program, the manner in which these experiences are handled is up to you. In some cases, you may have all students working with materials and resources individually; in others, you may choose to use small group settings. The latter encourages the development of social skills and enables all students to be active in the learning process; it also means less cost in terms of materials and equipment. Again, classroom management is left up to you, since it is the teacher who ultimately determines how the students in his/her care function best in the learning environment.

Planning Guidelines

Mathematics is a skills-based subject. In order to acquire these skills, students need to visit and revisit them over the course of the school year.

Hands-On Mathematics is organized into strand- or topic-focused units. This organization allows teachers to follow the development of concepts from introduction to mastery within a given grade level. To ensure that students have opportunities to develop their mathematical skills in all topics throughout the year, it is recommended that teachers address concepts from each of these units in every reporting period or school term. For example, teachers may choose to begin the year with the unit on Patterns and Relations (Unit 1) but should continue to develop students' skills in this area throughout the entire school year through review, continued practice, and new mathematical challenges. In the same way, although the unit on Number Operations (Unit 6) is presented last in the ***Hands-On Mathematics*** program, students should be provided with opportunities to review, practice, and investigate number operations throughout the school year.

Planning in this way gives students the time needed to solidify their understanding and, at the same time, helps to keep the concepts and vocabulary in the forefront throughout the year.

Note: Developing a year plan will ensure that topics are dealt with throughout the school year. Teachers can design the plan to meet their students' specific needs and fit into their school calendars. On the following two pages, a sample year-plan template is provided. Teachers can use the template to record the skills from each unit that they will teach during each month. There is also additional space for briefly noting other curriculum connections and themes that may relate to the overall math plan. Teachers can divide the bottom row of the chart according to the duration of the curriculum connection and related theme, which may be less than or greater than one month.

Note to Teachers

Hands-On Mathematics, Grade 3 is divided into two parts. Part 1 includes the Introduction to ***Hands-On Mathematics***, Unit 1: Patterning and Algebra, and Unit 2: Data Management and Probability. Part 2 includes Unit 3: Measurement, Unit 4: Geometry and Spatial Sense, Unit 5: Number Concepts, and Unit 6: Number Operations.

Assessment

The *Hands-On Mathematics* Assessment Plan

Hands-On Mathematics provides a variety of assessment tools that enable you to build a comprehensive and authentic daily assessment plan for your students. The assessment tools can be used to help teachers identify student achievement levels as outlined in the *Ontario Curriculum for Mathematics (2005)*. Please refer to page 19 for details on the achievement levels for mathematics.

Embedded Assessment

Assess students as they work by using the questions provided with each activity. These questions promote higher-level thinking skills, active inquiry, problem solving, and decision making. Anecdotal records and observations are examples of embedded assessment:

- **anecdotal records**: Recording observations during mathematics activities is critical in having an authentic view of a student's progress. The Anecdotal Record sheet, presented on page 20, provides the teacher with a format for recording individual or group observations.
- **individual student observations:** During activities when you wish to focus more on individual students, you may decide to use the Individual Student Observations sheet, found on page 21. This black line master provides more space for comments and is especially useful during conferencing, interviews, or individual student presentations.

Data collected from anecdotal records and individual student observations can help teachers identify the achievement levels of students according to their performance on given tasks.

Performance Assessment

Performance assessment is planned, systematic observation and assessment based on students actually doing a specific mathematics activity.

- **rubrics:** To assess students' performance on a specific task, rubrics are used in ***Hands-On Mathematics*** to standardize and streamline scoring. A sample rubric and a black line master for teacher use are included on pages 22 and 23. For any specific activity, the teacher selects four criteria that relate directly to the learning expectations for the specific activity being assessed. Students are then given a checkmark point for each criterion accomplished to determine a rubric score for the assessment from a total of four marks. These rubric scores can then be transferred to the Rubric Class Record on page 24. The rubrics have been designed to complement the achievement levels, as a four-point scale is used to identify student performance.

Cooperative Skills

To assess students' ability to work effectively in a group, teachers must observe the interaction within these groups. A Cooperative Skills Teacher Assessment sheet is included on page 25 for teachers to use while conducting such observations.

Student Self-Assessment

It is important to encourage students to reflect on their own learning in mathematics. For this purpose, teachers will find a Student Self-Assessment sheet on page 26, as well as a Cooperative Skills Self-Assessment sheet on page 27.

Unit 1

Patterning and Algebra

Introduction

Mathematics is the study of patterns and relations. When students begin to recognize and explore the patterns that are inherent in mathematics, it becomes easier for them to understand the relationships among different mathematical concepts. Students need opportunities to discover and explore both patterns that occur in everyday life as well as those revealed through calculators and computers.

Note: Although the Data Management and Probability strand of the *Ontario Curriculum for Mathematics (2005)* addresses sorting and classifying, most sorting skills are prerequisites for patterning. Accordingly, sorting activities are included in this unit on patterning and algebra. As well, the learning expectations that focus on "Expressions and Equality" (Patterning and Algebra strand) are addressed in Unit 6 of ***Hands-On Mathematics***, Number Operations, because of the connection between these expectations and number operations skills and concepts.

In this unit, students learn to recognize, describe, extend, and create patterns using real objects, mathematical materials, and numbers. Students first learn about patterns by identifying similarities and differences as they sort. Students explore various sorting activities at the beginning of the unit, learning to identify, describe, and classify objects by their attributes. As they start to understand the relationships between objects, students can begin making predictions about patterns. They then proceed to the recognition of visual patterns, auditory patterns, and patterns involving the sense of touch. From recognition, students progress to pattern extension, translation of patterns to other modes, and finally to the creation of their own patterns.

Students learn to create various forms of patterns in this unit including patterns using objects, geometric shapes, pictures, numbers, sounds, "touch" actions (for example, tapping), and physical actions (clapping, jumping, and so on). Students should be exposed to all different forms of patterning and should develop skills in transferring patterns from one form to another.

Teachers should also expose students to a wide variety of activities and play with patterns of all kinds including those from different cultures such as the patterns in ancient number systems like Roman numerals. These should consist of linear patterns, symmetrical patterns, repeating patterns, and increasing/decreasing patterns.

Mathematics Vocabulary

Students must learn to recognize and understand the mathematical vocabulary related to the patterns and relations unit. A "mathematics word wall" is a valuable reference for students for displaying new vocabulary. Dedicate a classroom bulletin board to your word wall, and display the letters of the alphabet along the top of the bulletin board. Use index cards to record math vocabulary introduced in each lesson, and place these on the board under the appropriate letter of the alphabet. Encourage students to refer to the math word wall during activities and while doing written tasks.

Throughout this unit, teachers should use, and encourage students to use, vocabulary such as: *attribute, sort, classify, set, similarities, differences, less, more, fewer, shape, pattern, element, term, repeat, increasing, decreasing, Venn diagram, Carroll diagram, intersection, compare, extend, vertical, horizontal, row, column, diagonal,* and *core*.

2 Describing Attributes

Background Information for Teachers

An *attribute* is a characteristic or quality of something or someone that can be used for sorting purposes. In order to sort into complex groupings, grade three students will need to review what an attribute is. The activities in this lesson encourage students to use language to describe buttons, attribute blocks, and other objects. Students review sets, attributes, sorting rules, and discriminating similarities and differences.

As noted previously, sorting activities are included in this unit, rather than in the Data Management and Probability unit, to help develop in students the skills that are prerequisite for patterning.

Materials

- *Sorting* (*Math Counts*), a book by Henry Pluckrose
- variety of sets of objects (beans, shells, keys, school supplies, small toys, attribute blocks, and so on)
- "Button Fun," a poem by Dakshana Bascaramurty (included. Print the poem onto chart paper. Be sure to print large, and leave space between each line of the poem.) (1.2.1)
- collection of buttons (be sure there is variation in size, colour, shape, number of holes, and material from which buttons are made. You will need twenty to thirty buttons for each working group of students.)
- clear tape
- attribute blocks (you will need several blocks for each pair of students)

Activity: Part One: Describing Attributes

Read the book, *Sorting* (*Math Counts*) together with students. While reading, be sure to stop frequently, asking students to describe in detail the objects on each page. Discuss the similarities and the differences in the sorted sets, and review the questions asked in the book. Make a list of sorting rules to keep track of the attributes used in the book (colour, size, shape, function, length, type, location).

Have students sit in a circle. Place a variety of objects (beans, shells, keys, school supplies, small toys, attribute blocks, and so on) in the centre of the circle, and play "I Spy" with students. Provide three or four clues (both positive and negative) about one of the objects, and have students determine which object you are describing. For example, referring to a toy car, you might say:

- I spy an object that is red.
- It is not used for drawing.
- It has wheels.
- What object am I thinking of?

Pause after each clue, allowing one or two guesses from students before providing another clue. When a student guesses correctly, ask him/her:

- How did you know you were naming the correct object?
- Which other objects are red, are not used for drawing, or have wheels?
- How else could I describe my object? (small, a toy, a type of transportation)

Repeat the activity, but have a student give the clues. Remind the student to describe the object in as many different ways as possible using both positive and negative clues.

When students become familiar with the game, have them play it in small groups. After each round, the student who guessed correctly gives clues about a new object in the next round.

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Activity: Part Two: Attribute Poems

Display for students the poem "Button Fun" (1.2.1), which you have printed on chart paper. Read through the poem together as a class, and have students identify and highlight all the words that are used to describe buttons.

Divide the class into working groups, and provide each group with some clear tape as well as twenty to thirty buttons to use in an attribute hunt. Have students in each group search for a button in their collection that has one of the attributes mentioned in the poem. When they find a button with a matching attribute, have one student from the group tape their button just above or below that attribute on the poem. Have students continue the attribute hunt until each group has found six buttons to match six different attributes listed in the poem.

Divide the class into pairs of students, and distribute a copy of Activity Sheet A (1.2.2) to each student. Present sets of objects (beans, shells, keys, school supplies, small toys, and so on) to students, and have each pair choose a set and make a list of all the attributes they can think of to describe the objects in their set. For example, attributes to describe the set of keys could include: big, small, silver, gold, coloured, shiny, dull, new, old.

Once pairs have completed their lists, have each pair use their list to create their own attribute poem. Later, have each pair share their attribute poem with the rest of the class. Or, display the poems throughout the classroom, or bind them into a class attribute poem book.

Note: Save the attribute lists from Activity Sheet A (1.2.2) for use in an upcoming lesson.

Activity Sheet A

Directions to students:

Note: This is a two-page activity sheet.

Make a list of all the different attributes you can think of to describe your set of objects. Record one attribute in each box. Record the title of your set of objects in the blank at the top of the sheet. Once you have completed your list, use it to create your own attribute poem (1.2.2).

Activity: Part Three: Similarities and Differences

Have students sit in a circle. Hold up an attribute block for students to see. Ask students to describe as many of the block's attributes as they can including number of sides, thickness, colour, and size. For example:

- The block is yellow.
- It has six sides.
- It is thin.
- It is large.

Place the attribute block in the centre of the circle. Give each student his/her own attribute block, and tell students they will now build a "one-difference train." Ask students to look carefully at their blocks. Ask:

- Does anyone have a block that is different from my block by (because of) only one attribute?

Have any student who replies positively explain how his/her block is the same as yours and describe the one attribute that is different. For example:

- My block and your block are both yellow.
- They both have six sides.
- They are both large.
- My block is thick and your block is thin.

2

If the student's block differs from yours by only one attribute, have the student place it next to your block in the centre of the circle. Then, ask:

- Does anyone have a block that is different from (student)'s block by only one attribute?

Repeat the process three or four more times until students understand the concept.

Once the one-difference train is five or six attribute blocks long, remove all blocks except the original one from the centre of the circle. Redistribute one attribute block to each student who had added his/her block to the one-difference train. Tell students they will now build a "two-difference train." Ask:

- Who has an attribute block that is different from my block in exactly two ways?

Repeat the same process as before with students describing similarities and differences to ensure that the block fits onto the train.

Divide the class into pairs of students, and distribute several attribute blocks to each pair. Have the pairs create their own two-difference trains, with each block different from the one next to it in two ways.

Distribute Activity Sheet B (1.2.3) to students. Using the two-difference trains they created with attribute blocks, have students record all four attributes for each block by printing them in the boxes. Ensure that they record the attributes for each block in the same order: colour, sides (number of sides), size, and thick/thin. Once they have recorded attributes for each block, have students draw lines between corresponding attributes from one box to the next. If the attribute is the same (blue, blue) the line should be solid. If the attribute is different the line should be dotted, as in the following example:

Then, challenge students to use attribute blocks to make three-difference trains, with each block different from the one next to it in three ways. Again, ask students to record all four attributes for each block by printing them in the boxes on the activity sheet and then drawing solid lines between similar attributes and dotted lines between different attributes.

Activity Sheet B

Directions to students:

Using the two-difference train you created with attribute blocks, record all four attributes for each block by printing them in the boxes. Be sure you record the attributes for each block in the same order: colour, sides (number of sides), size, and thick/thin. Draw solid lines between similar attributes from one box to the next, and draw dotted lines between different attributes from one box to the next. Then, try using attribute blocks to make a three-difference train, with each block different from the one beside it in three ways. Record all four attributes for each block by printing them in the boxes. Draw solid lines between similar attributes and dotted lines between different attributes (1.2.3).

Problem Solving

Carl went shopping to buy a present for his sister. He considered a doll, a stuffed puppy, some coloured blocks, and a music box. The gift Carl chose does not make noise. It is soft. You cannot build with it. It has four legs. What did Carl buy for his sister?

Note: A reproducible master for this problem can be found on page 255.

2

Activity Centres

- Place a bag filled with small objects (beans, shells, keys, buttons, school supplies, small toys, plastic animals, and so on) at an activity centre along with scrap paper and pencils. Have students choose two objects from the bag and record three similarities and three differences between the objects. As a variation, have two or more students compete to see who can list the most similarities/differences between the two objects.
- Have groups of two or more students play Mattel's TriBond Junior. The object of this game is to find the commonality between three objects. For example, what do a turtle, a nut, and an egg all have in common? (They all have shells.) Or, what do a shark, a comb, and a saw all have in common? (They all have teeth.)

  Or, make up your own questions that are related to other units of study. For example:

  - What do stamens, pistils, and petals have in common? (They are all parts of a flower.)
  - What do beams, braces, and joints all have in common? (They all help to support structures.)

  After a little practice, students can try making up their own clues.
- Have groups of two to four students play Ravensburger's Guess My Name Game. One player gives clues to describe an object, while the other players try to be the first to guess what the object is. For example:

  - I live in the water.
  - I am covered with feathers.
  - I say "quack."
  - What am I?

Extensions

- Add the word *attribute* to your math word wall.
- Divide the class into pairs of students, and provide each pair with a set of twelve to sixteen small objects as well as scrap paper and pencils. Have each pair place their set of objects between them and play What Is My Object? Ask player *A* to select one of the objects and record which one he/she has chosen without player *B* seeing. Then, have player *B* try to guess which object player *A* chose by asking "yes or no" attribute questions. For example, a set of sixteen buttons with different attributes is placed between the partners, so that both partners can see all of the buttons. Player *A* selects a red, plastic, square button with four holes. He records his choice to show player *B* at the end of the game. Player *B* asks:

  - Is your button round?
  - Is your button blue?
  - Is your button made of plastic?

  Examples of questions that are not suitable (because they do not elicit "yes" or "no" answers) include:

  - What colour is your button?
  - How many holes does your button have?

  Tell students to keep a tally of the number of questions asked. Have students play several times so they can begin to develop strategies to guess the object in as few questions as possible.
- Read with students *Hannah's Collections*, a book by Marthe Jocelyn. Have each student bring a collection of his/her own to school and describe it to the class.

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- Have students play Fripple Place (available through the Winnipeg School Division freeware suite). Players examine the clues and use deductive reasoning to sort the Fripples by their attributes and then place them into the correct rooms.
- Have students play one of two interactive logic games found at the Learner.Org Teacher's Lab Patterns in Mathematics website (Annenberg/CPB) at http://www.learner.org/teacherslab/math/patterns/logic.html:
  - In Guess My Button, the computer secretly chooses a button, and players must figure out which one it is.
  - In People Patterns, players must figure out who should come next in a sequence of people.

Assessment Suggestion

Observe students as they participate in the attribute activities. Assess each student's ability to:

- use a variety of attributes (size, colour, function, material, and so on) to describe objects
- distinguish similarities and differences between objects
- use attribute blocks to complete a two-difference train
- use attribute blocks to complete a three-difference train

List these criteria on the Rubric sheet, found on page 23, and record your observations.

Button Fun

By Dakshana Bascaramurty

I have fun with buttons
That have fallen off my clothes,
I sort them into columns
And I sort them into rows.

Metallic, dull or shiny,
Circular or square,
Buttons come in fours or threes
And sometimes just in pairs.

Brown, black, or white buttons
Silver, brass, or gold,
Big buttons, little buttons,
Buttons – new or old.

I like to put them into sets
By colour, shape, or size,
In a button-sorting contest
I would surely win first prize!

Date: _____ **Names:** _____

_____

Using Attributes to Write a Poem

Attribute List for _____

Date: _____ **Names:** _____

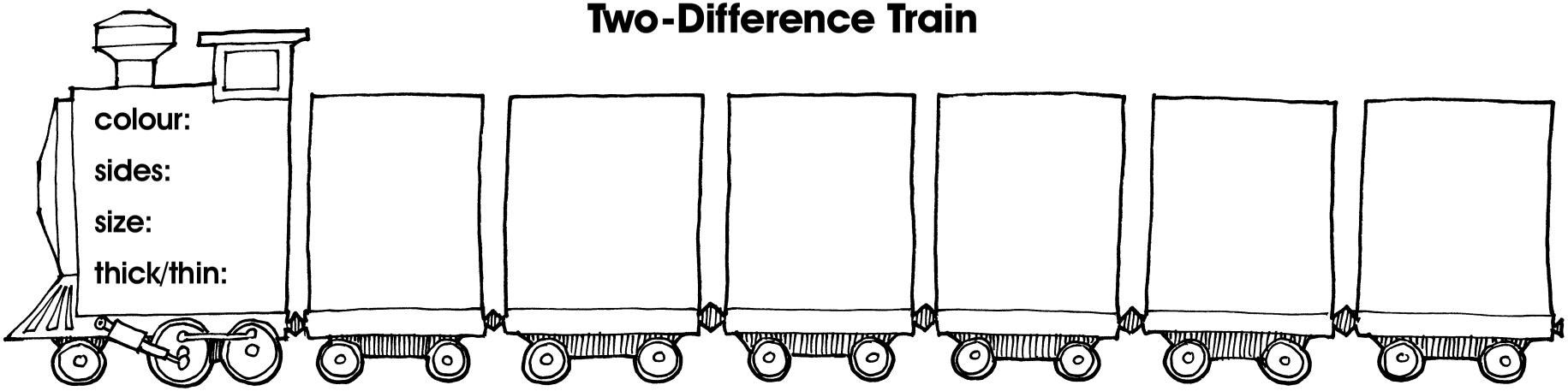
_____

Title: _____

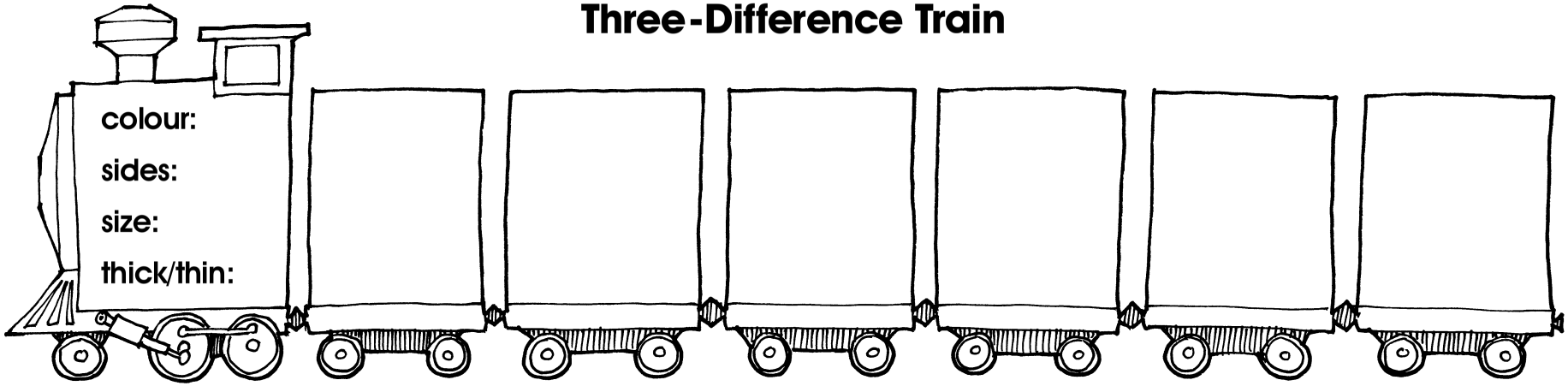
Date: _____ Name: _____

Two- and Three-Difference Trains

Two-Difference Train



Three-Difference Train



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