hands-on mathematics
Grade 2

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Introduction to *Hands-On Mathematics*

**Program Introduction**

*Hands-On Mathematics* focuses on developing students’ knowledge, skills, and attitudes through active inquiry, problem solving, and decision making. Throughout all activities, students are encouraged to explore, investigate, and ask questions in order to heighten their own curiosity about and understanding of the world of mathematics.

**Program Principles**

1. Effective mathematics programs involve students actively building new knowledge from experience and prior knowledge.

2. The development of students’ understanding of concepts, flexibility in thinking, reasoning, and problem-solving skills/strategies form the foundation of the mathematics program.

3. From a young age, children are interested in mathematical ideas. This interest must be maintained, fostered, and enhanced through active learning.

4. Mathematics activities must be meaningful, worthwhile, and relate to real-life experiences.

5. The teacher’s role in mathematics education is to actively engage students in tasks and experiences designed to deepen and connect their knowledge. Children learn best by doing, rather than by just listening. The teacher, therefore, should focus on creating opportunities for students to interact, in order to propose mathematical ideas and conjectures, to evaluate their own thinking and that of others, and to develop mathematical reasoning skills.

6. Mathematics should be taught in correlation with other school subjects. Themes and topics of study should integrate ideas and skills whenever possible.

7. The mathematics program should encompass, and draw on, a range of educational resources, including literature and technology, as well as people and places in the local community.

8. Assessment of student learning in mathematics should be designed to focus on performance and understanding, and should be conducted through meaningful and varied assessment techniques carried on throughout the modules of study.

**The Big Ideas of Mathematics**

In order to achieve the goals of mathematics education and to support lifelong learning in mathematics, students must be provided with opportunities to encounter and practice critical mathematical processes. These processes are as follows:

**Communication**

Students need to be given opportunities to communicate their mathematical ideas through the use of oral language, reading and writing, diagrams, charts, tables, and graphs.

For example:

```
What is adding
adding is putting
how numbers together
and counting same like
1+1 makes 2 or 2
2+3 makes 5 or 5
and that's
what adding is
```
As another example:
Show different ways the apples could be put into bowls.
There are 16 apples.
The apples are in bowls.
Each bowl has the same number of apples in it.

Connections
Teachers need to ensure that connections are made between the various modules, or strands, of the curriculum. It is also important to make connections between concrete, pictorial, and symbolic mathematical representations. Further, concepts and skills should be connected to everyday life and to other curricular areas. For example:

Use words and numbers to tell about the pictograph.
1. There are 4 apples.
2. There are 2 bananas.
3. There are 5 oranges.
4. There are 11 fruits altogether.
5. Apples and oranges equal 9.
6. Oranges and bananas equal 7.
8. There are the most oranges.
9. There are the least bananas.

The Hands-On Mathematics program offers many ways to connect mathematics to children’s literature through the use of storybooks in lessons. Many lessons begin with a springboard activity, which involves reading a book that relates to the concepts focused on in the lesson. The books suggested are well known and are usually available in school or public libraries.
Before beginning each module, teachers are encouraged to review the lessons, as well as the list of suggested children’s books, and acquire the recommended books. If, however, a suggested book is not available, you can substitute another similar book, or you can simply move ahead to the next part of the lesson.

**Mental Math and Estimation**

Mental math is a process necessary to many everyday experiences. Students need extensive exposure to activities that encourage them to solve problems mentally, without the use of concrete objects or paper/pencil supports. Students should be encouraged regularly to estimate quantities and measurements. Estimation encourages them to take risks, use background knowledge, and learn from the process. For example:

- **Mental Math:** Students respond quickly to questions phrased in a variety of ways:
  - Double 4
  - Half of 6
  - Two 5s
  - You roll double 3. What’s your score?
  - How many shoes in 2 pairs?

Suggestions for Mental Math activities can be found on pages 28 through 29.

- **Estimation:** Estimate whether there are enough spoons to go with these forks.

Now, check. Are there too many or too few spoons?

**Problem Solving**

Students are exposed to a wide variety of problems in all areas of mathematics. They explore a variety of methods for solving and confirming their solutions to both routine and non-routine problems. They should also be encouraged to find multiple solutions for problems and to create their own problems.

- **Routine Problems:** These are problems in which the way to a solution is immediately evident. The solution generally involves one or two arithmetic operations. For example:
  - When the cow jumped over the moon, she counted 75 craters in all. If one half of the moon had 34 craters, how many craters were on the other half?

- **Non-Routine Problems:** These problems are more challenging for students. Upon first reading, the path to a solution is not immediately evident. Students draw on a bank of strategies (teacher-presented and student-developed) to solve the problem. Some of these problems can have more than one solution/answer. Others can be solved using a variety of strategies. For example:
  - There are 5 flowers in the basket. If the flowers are pink, yellow, and lavender, how many of each colour could there be? Find all of the combinations. (Draw a diagram, use materials, act it out.)
  - Ed does sit-ups daily. On Monday, he did 20 sit-ups. On Tuesday, he did 30. He did 50 sit-ups on Wednesday, and 90 on Thursday. If he continues this pattern, how many sit-ups will Ed do on Saturday? (Look for a pattern.)
A huge frog ate 140 big bugs in 5 days. Each day, it ate 8 more bugs than it did on the previous day. How many bugs did the frog eat each day? (Guess and test.)

Tina makes snow people out of 3 snowballs. Kim uses 2 snowballs for her snow people. The girls made 5 snow people altogether. They used 13 snowballs. What did their snow people look like? (Draw a diagram.)

Reasoning
Mathematical reasoning involves informal thinking, conjecturing, and validating. Students should be encouraged to justify their solutions, thinking processes, and hypotheses. Good reasoning is as important as finding correct answers, so students need many opportunities to think about, describe orally, and record their mathematical activities and ideas. For example:

Logic Problems:
(a) Monkey, Frog, Caterpillar, and Alligator are growing onions, carrots, corn, and potatoes in their garden. Follow the clues to match the animals with the vegetables they are growing.

1) The vegetables that Monkey, Frog, and Caterpillar are growing all grow underground.
2) The vegetable that Monkey is growing makes Frog cry.
3) The vegetable that Frog is growing can be made into chips.

(b) Snake, Caterpillar, and Elephant had a swinging contest. They each took a turn and then decided who won first place, who won second place, and who won third place.

1) Snake said, “Of course Elephant was higher than I was. He used his trunk to help.”
2) “I could hardly get my swing to move. How did you get going, Snake?” asked Caterpillar.

Technology
The use of calculators is recommended, to enhance problem solving and to encourage discovery of number patterns. However, calculators must not replace the development of students’ number concepts and skills. Other technologies such as computer software and web sites can provide valuable resources for students and teachers.
**Visualization**

These are the mental images needed to develop concepts and understand procedures. Visualizations help students clarify their understanding of mathematical ideas. For example:

- Show all you know about the number 25. Use pictures, diagrams, and words in your answer.

```
[\[25\]]

five groups of 5
3 3 3 3 3
2 digit number
odd number
20 + 5
twenty-five
[\[\[\]]

2 tens 5 ones
20 + 5
25 \& is a quarter
1 ten 15 ones
```
Note: These learning outcomes have been established by the Western and Northern Canadian Protocol (WNCP) as outlined in the document the Common Curriculum Framework for K–9 Mathematics (May, 2006).

Processes Key
[C] Communication [PS] Problem Solving
[CN] Connections [R] Mathematical Reasoning
[ME] Mental Mathematics and Estimation
[T] Technology [V] Visualization

MODULE 1: PATTERNS AND RELATIONS

General Outcomes
■ Use patterns to describe the world and solve problems
■ Represent algebraic expressions in multiple ways

Specific Outcomes
1. Demonstrate an understanding of repeating patterns (three to five elements) by:
   ■ describing
   ■ extending
   ■ comparing
   ■ creating patterns using manipulatives, diagrams, sounds, and actions
   [C, CN, PS, R, V]
2. Demonstrate an understanding of increasing patterns by:
   ■ describing
   ■ reproducing
   ■ extending
   ■ creating patterns using manipulatives, diagrams, sounds and actions (numbers to 100)
   [C, CN, PS, R, V]
3. Demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100)
   [C, CN, R, V]
4. Record equalities and inequalities symbolically using the equal symbol or the not equal symbol [C, CN, R, V]

MODULE 2: STATISTICS AND PROBABILITY

General Outcomes
■ Collect, display, and analyze data to solve problems

Specific Outcomes
1. Gather and record data about self and others to answer questions [C, CN, PS, V]
2. Construct and interpret concrete graphs and pictographs to solve problems [C, CN, PS, R, V]

MODULE 3: SHAPE AND SPACE

General Outcomes
■ Use direct or indirect measurement to solve problems
■ Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them

Specific Outcomes
Measurement
1. Relate the number of days to a week and the number of months to a year in a problem-solving context [C, CN, PS, R]
2. Relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass (weight) [C, CN, ME, R, V]
3. Compare and order objects by length, height, distance around and mass (weight) using non-standard units, and make statements of comparison [C, CN, ME, R, V]
4. Measure length to the nearest non-standard unit by:
   ■ using multiple copies of a unit
   ■ using a single copy of a unit (iteration process)
   [C, ME, R, V]
5. Demonstrate that changing the orientation of an object does not alter the measurements of its attributes [C, R, V]
3-D Objects and 2-D Shapes
6. Sort 2-D shapes and 3-D objects using two attributes, and explain the sorting rule [C, CN, R, V]
7. Describe, compare and construct 3-D objects including:
   - cubes
   - spheres
   - cones
   - cylinders
   - pyramids [C, CN, R, V]
8. Describe, compare and construct 2-D shapes including:
   - triangles
   - squares
   - rectangles
   - circles [C, CN, R, V]
9. Identify 2-D shapes as parts of 3-D objects in the environment [C, CN, R, V]

MODULE 4: NUMBER CONCEPTS
Note: The Number strand of The Common Curriculum Framework for K–9 Mathematics (May, 2006) has been divided into two separate modules: Number Concepts (module 4) and Number Operations (module 5).

General Outcome
- Develop number sense
Specific Outcomes
1. Say the number sequence from 0 to 100 by:
   - 2s, 5s and 10s, forward and backward, using starting points that are multiples of 2, 5 and 10 respectively
   - 10s using starting points from 1 to 9
   - 2s starting from 1 [C, CN, ME, R]
2. Demonstrate if a number (up to 100) is even or odd [C, CN, PS, R]
3. Describe order or relative position using ordinal numbers (up to tenth) [C, CN, R]
4. Represent and describe numbers to 100, concretely, pictorially and symbolically [C, CN, V]
5. Compare and order numbers up to 100 [C, CN, R, V]
6. Estimate quantities to 100 by using referents [C, ME, PS, R]
7. Illustrate, concretely and pictorially, the meaning of place value for numerals to 100 [C, CN, R, V]

MODULE 5: NUMBER OPERATIONS
General Outcome
- Develop number sense
Specific Outcomes
1. Demonstrate and explain the effect of adding zero to or subtracting zero from any number [C, R]
2. Demonstrate an understanding of addition (limited to 1 and 2-digit numerals) with answers to 100 and the corresponding subtraction by:
   - using personal strategies for adding and subtracting with and without the support of manipulatives
   - creating and solving problems that involve addition and subtraction
   - explaining that the order in which numbers are added does not affect the sum
   - explaining that the order in which numbers are subtracted may affect the difference [C, CN, ME, PS, R, V]
3. Apply mental mathematics strategies to determine basic addition facts to 18 and related subtraction facts. This includes mental math strategies such as:
   - using doubles
   - making 10
   - one more, one less
   - two more, two less
   - addition for subtraction [C, CN, ME, R, V]
Program Implementation

Program Resources

*Hands-On Mathematics* is arranged in a format that makes it easy for teachers to plan and implement.

Modules comprise the selected topics of study for the grade level, organized into lessons. The modules relate directly to the outcomes identified on pages 6 and 7, which complement those established by the Western and Northern Canadian Protocol (WNCP) and outlined in *The Common Curriculum Framework for K-9 Mathematics* (May 2006).

The introduction to each module summarizes the general goals for the module and provides background information for teachers. Each module begins with a list of books for students that relate to the module; a list of related websites (for all modules combined) can also be found on pages 52 and 53.

Modules are organized into lessons, based on the outcomes.

**Note:** This does not imply that a lesson can be covered in only one lesson period; many will carry over several lesson periods.

The lessons are arranged in the following format:

**Background Information for Teachers:**

Some lessons provide teachers with the basic mathematical knowledge they will need to present the activities. This information is offered in a clear, concise format, and focuses specifically on the topic of study.

**Materials:** A complete list of materials required to conduct the main activity or activities is provided. It includes classroom materials, equipment, and visuals. The quantity of materials required will depend on how you conduct activities and whether students are working individually or in groups.

**Activity/Activities:** This section details a step-by-step procedure, including higher-level questioning techniques and suggestions for encouraging active inquiry and discussion.

**Activity Sheets:** Reproducible activity sheets have been designed to correlate with the specific outcomes of the activity or activities. Many of these are used during the activity to record results of investigations. Others are used as follow-up to the in-class activities. Students may work independently on these sheets, in small groups, or you may choose to read through them together and complete them in a large group setting. Activity sheets can also be made into overheads or large experience charts. Since it is also important for students to learn to construct their own charts and recording formats, these activity sheets can be used by the teacher as examples of ways to record and communicate ideas about an activity. Students can then create their own sheets rather than use the ones provided.

**Note:** Activity sheets are meant to be used only in conjunction with, or as a follow-up to, the hands-on activities. The activity sheets are not intended to be the mathematics lesson in itself or the sole assessment for the lesson.

**Next Step:** Some lessons, particularly in the Number Concepts module, include a section called “Next Step(s),” which guides teachers through a subsequent activity or sequence of activities to carry out with students, following developmentally from the preceding activity or activities. For example, in the main activities of lesson 10, Patterns and Relations module, students use balance scales and sets of objects to 20 to explore equality and inequality. Once students master this, the next step is to move to using sets of objects to 30, then to 50, and onward to 100 as students show mastery at each stage.
Problem Solving: Many lessons include suggestions for problem-solving activities that are directly related to the lesson’s outcomes. These problems may be presented orally, acted out with concrete objects, presented pictorially, or written out on chart paper. At the end of each module, many of these problems are presented again on black line masters. Teachers can copy these sheets onto overhead transparencies to present to students as daily problem-solving activities. Or, the masters can be copied for students and cut apart, problem by problem. Students can then paste the problems into their math journals or agendas for completion independently.

Extension: Many lessons include optional activities to extend, enrich, and reinforce the outcomes.

Activity Centre: Some lessons include independent student activities that focus on the outcomes.

Assessment Suggestions: Throughout each module, several suggestions are made for assessing student learning. Again, these assessment strategies focus specifically on the learning outcomes of a particular activity topic. In the next section of the Hands-On Mathematics program, assessment is dealt with in detail. Keep in mind that the suggestions made within activities are merely ideas to consider; you may use your own assessment techniques or refer to the other assessment strategies on pages 13 and 14.

Classroom Environment

The classroom setting is an important component of the learning process. An active environment – one that gently hums with the purposeful conversations and activities of students – indicates that meaningful learning is taking place. While studying a specific topic, the room should display related objects and materials, student work, pictures and posters, maps, graphs, and charts made during activities, and summary charts of important concepts taught and learned. These reinforce concepts and skills that have been stressed during mathematics activities.

Timelines

No two groups of students will cover topics and material at the same rate. Planning the duration of modules is the responsibility of the teacher. In some cases, the activities described will not be completed during one block of time and will have to be carried over. (Division of modules into “lessons” does not imply that they can be covered in only one lesson period. Many lessons will, in fact, carry over several lesson periods.) In other cases, you may observe that the students are especially interested in one topic, and you may choose to expand upon it. The individual needs of your students should be considered as there are no strict timelines involved in the Hands-On Mathematics program. It is important, however, to spend time on every module in the program so that students focus on all of the outcomes established for their grade level.

Classroom Management

Although active learning is emphasized throughout this program, the manner in which these experiences are handled is up to you. In some cases, you may have all students working with materials and resources individually; in others, you may choose to use small group settings. The latter encourages the development of social skills and enables all students to be active in the learning process; it also means less cost in terms of materials and equipment. Again, classroom management is left up to you, since it is the teacher who ultimately determines how the students in his/her care function best in the learning environment.
Planning Guidelines

Mathematics is a skills-based subject. In order to acquire these skills, students need to visit and revisit them over the course of the school year.

*Hands-On Mathematics* is organized into strand- or topic-focused modules. This organization allows teachers to follow the development of concepts from introduction to mastery within a given grade level. To ensure that students have opportunities to develop their mathematical skills in all topics throughout the year, it is recommended that teachers address concepts from each of these modules in every reporting period or school term. For example, teachers may choose to begin the year with the module on Patterns and Relations (Module 1) but should continue to develop students' skills in this area throughout the entire school year through review, continued practice, and new mathematical challenges. In the same way, although the module on Number Operations (Module 5) is presented last in the *Hands-On Mathematics* program, students should be provided with opportunities to review, practice, and investigate number operations throughout the school year.

Planning in this way gives students the time needed to solidify their understanding and, at the same time, helps to keep the concepts and vocabulary in the forefront throughout the year.

**Note:** Developing a year plan will ensure that topics are dealt with throughout the school year. Teachers can design the plan to meet their students’ specific needs and fit into their school calendars. On the following two pages, a sample year plan template is provided. Teachers can use the template to record the skills from each module that they will teach during each month. There is also additional space for briefly noting other curriculum connections and themes that may relate to the overall math plan. Teachers can divide the bottom row of the chart according to the duration of the curriculum connection and related theme, which may be less than or greater than one month.
Assessment

The Hands-On Mathematics Assessment Plan

*Hands-On Mathematics* provides a variety of assessment tools that enable you to build a comprehensive and authentic daily assessment plan for your students.

**Embedded Assessment**
Assess students as they work by using the questions provided with each activity. These questions promote higher-level thinking skills, active inquiry, problem solving, and decision making. Anecdotal records and observations are examples of embedded assessment:

- **anecdotal records**: Recording observations during mathematics activities is critical in having an authentic view of a student’s progress. The Anecdotal Record sheet, presented on page 15, provides the teacher with a format for recording individual or group observations.

- **individual student observations**: During activities when you wish to focus more on individual students, you may decide to use the Individual Student Observations sheet, found on page 16. This black line master provides more space for comments and is especially useful during conferencing, interviews, or individual student presentations.

**Performance Assessment**
Performance assessment is planned, systematic observation and assessment based on students actually doing a specific mathematics activity.

- **rubrics**: To assess students’ performance on a specific task, rubrics are used in *Hands-On Mathematics* to standardize and streamline scoring. A sample rubric and a black line master for teacher use are included on pages 17 and 18. For any specific activity, the teacher selects four criteria that relate directly to the learning outcomes for the specific activity being assessed. Students are then given a checkmark point for each criterion accomplished to determine a rubric score for the assessment from a total of four marks. These rubric scores can then be transferred to the Rubric Class Record on page 19.

**Cooperative Skills**
To assess students’ ability to work effectively in a group, teachers must observe the interaction within these groups. A Cooperative Skills sheet is included on page 20 for teachers to use while conducting such observations.

**Student Self-Assessment**
It is important to encourage students to reflect on their own learning in mathematics. For this purpose, teachers will find a Student Self-Assessment sheet on page 21, as well as a Cooperative Skills Self-Assessment sheet on page 22.

In addition, a Math Journal sheet is found on page 23. Teachers can copy several sheets for each student, cut them in half, add a cover, and bind the sheets together. Students can then create title pages for their own journals. For variety, you may also have students use the blank back sides of each page for other reflections. For example, have students draw or write about:

- numbers we see in our homes
- numbers we see in the community
- numbers in books
- favourite math activities
- math-related book reports
- new math terminology
Students will also reflect on their own learning through writing in their math journals.

For young students, self-assessment is best done through oral discussion, reflecting on activities done in class. These conversations can occur as individual student conferences, small-group discussions, or as whole-class discussions.

**Portfolios**

Select, with student input, work to include in a mathematics portfolio, or in a mathematics section of a multi-subject portfolio. This can include activity sheets, patterning samples, graphs, charts, as well as other written material.

Use the portfolio to reflect the student’s progress in mathematics over the course of the school year. Black line masters are included to organize the portfolio (Portfolio Table of Contents on page 24 and Portfolio Entry Record on page 25). Students can be assisted in completing these sheets by having an adult scribe for them.

**Note:** Throughout each module of *Hands-On Mathematics*, suggestions for assessment are provided for several lessons. It is important to keep in mind that these are merely suggestions. Teachers are encouraged to use the assessment strategies presented in a wide variety of ways and to ensure that they build an effective assessment plan using these assessment ideas as well as their own valuable experience as educators.
Module 2

Statistics and Probability
Introduction

In this module, students collect and organize information/data using various strategies. Students also interpret this data, draw conclusions from it, and then extend the information.

The primary goal for this module is to strengthen and enhance students’ abilities to collect, record, and interpret data, using a variety of data management tools. A second goal is to enhance students’ mathematical vocabulary and language skills, in the area of data management, through both verbal and written communication.

Throughout the module, students will be provided with opportunities to construct and interpret concrete graphs, pictographs, charts, tallies, and glyphs.

Mathematics Vocabulary

Throughout this module, teachers should use, and encourage students to use, vocabulary such as: data, information, survey, tally, chart, Venn diagram, concrete graph, pictograph. Also, consider adding these terms to your classroom Math Word Wall as they are introduced in each lesson.
7

Sorting and Graphing Apples

Materials
- a variety of apples (You will need at least three varieties/colours of apples: one red, one green, and one gold apple. Have enough apples for each student to taste a piece of each variety.)
- knife
- paper plates
- pencils
- apple templates (included. Photocopy, and cut apart along dotted lines. You will need two paper apples per student.) (2.7.1)
- crayons, pencil crayons, or markers
- scissors
- chart paper
- markers
- overhead transparency of Activity Sheet A (2.7.2)
- overhead projector
- non-permanent overhead markers
- two Hula-Hoops (or two long pieces of string, formed into loops)
- large sheet of Bristol board
- index cards

Activity: Part One: Sorting Apples
Display the collection of apples, and have students examine them. Ask:
- How are these apples the same?
- How are they different?
Discuss the colour and size of the various apples. Ask:
- Can you put the apples in order from smallest to largest? Lightest to heaviest?
As a class, put the apples in order by these criteria. Ask:
- How can we sort the apples?
Brainstorm sorting rules, and have students sort the apples in a variety of ways. Discuss sorting rules and attributes (colour, shape, stem/no stem, texture, and so on).

Activity: Part Two: Graphing Apples on a Pictograph
Safety Note: In this activity, students taste apples. Be aware of any student allergies before doing the activity.
Select three varieties/colours of apples to use for this graphing activity. Display the apples, and discuss their names and features. Explain to students that they will each taste the three varieties of apples and pick their favourite. Ask:
- Which apple do you think you will like the best?
Provide each student with a paper plate. Ask:
- How could we divide each plate into three equal parts?
Discuss students’ ideas, using the opportunity to talk about fractions. Then, have students draw pencil lines on their plates to divide them into three sections, one for each type of apple. Have students label the sections with the names of the three varieties of apples.
Now, cut up the apples so that there is one piece of each variety for each student. Leave one apple of each variety intact to serve as an example.
Safety Note: Be sure students wash their hands before and after handling the apples.
Distribute the apple pieces, and have students place them on the appropriately labelled sections on their paper plates. Then, have students sample each variety of apple and decide which kind they like best.
When students have decided on their favourites, give each student a blank apple template (2.7.1), and have students colour their apples the same
colours as their favourite types of apples. Also, have students write their own names on their paper apples, and cut the apples out.

**Note:** Encourage students to colour their paper apples as accurately as possible, looking closely at the colours on the real apples and trying to replicate them. This may mean having two shades or colours on one paper apple, since many apples are not one solid colour.

Explain to students that they will now graph the results of this activity to show what type of apple each student in the class likes best. With student input, draw a large pictograph on chart paper. Have students place their coloured paper apples in the correct locations on the graph, as in the example below:

### Activity: Part Three: Sorting Apples on a Venn Diagram

Place two Hula-Hoops or string circles on a large sheet of Bristol board to create an intersecting Venn diagram. Use two index cards to label the circles “Red” and “Green,” as in the following example:

Provide each student with another apple template and again have each student colour his/her paper apple the colours of the apple variety he/she likes best. Have students cut out their paper apples and print their own names on them.

Now, have students look carefully at the Venn diagram. Ask:

- If your apple is completely green, where on the Venn diagram should you put your apple? What if your apple is completely red?
- Where should you put an apple that is both red and green?
- Where should you put a yellow apple?

Have students put their paper apples in the correct location on the Venn diagram.

### Activity: Part Four: A Survey About Apples

Explain to students that they will now complete surveys to find out the different ways students like to eat apples. Brainstorm with students for ideas of how apples can be prepared. For example: apple pie, apple crisp, apple sauce, caramel apple, baked apple, dried apple, apple chips.
Display the overhead copy of Activity Sheet A (2.7.2). Explain to students that they must first choose three “ways” of eating apples for their surveys and print these at the tops of their charts. Then, they will survey their classmates to find out which of those three ways of eating apples each student prefers.

Remind students that they need to record the names of students they question, to ensure that they survey the entire class. When they complete their surveys, students will also need to calculate the total number of names they have recorded in each column of their charts.

Allow students time to circulate and collect data.

**Activity Sheet A**

**Directions to students:**

Choose three “ways” of eating apples, and record these on your chart. Survey your classmates to find out which of these ways of eating apples is each student’s favourite (2.7.2).

**Problem Solving**

An average person eats 3 apples per week. How many apples would an average person eat in a month? How many apples would 10 average people eat in a week? How many apples would a class of students the size of yours eat in a week?

**Note:** A reproducible master for this problem can be found on page 214.

**Extensions**

- Add the terms *Venn diagram*, *data*, and *information* to your classroom Math Word Wall.
- Plan a field trip to a grocery store. Visit the produce department to do further research on apple varieties. Prepare a list of questions to ask the produce manager.
- Collect apple recipes, and create a class cookbook. Test each recipe by making it in class.

**Assessment Suggestion**

Have students record in their math journals what they learned during the sorting and survey processes. Use copies of the Math Journal sheet, found on page 23.
# Ways of Eating Apples Survey

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