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Program Implementation

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Hands-On Mathematics is arranged in a format that makes it easy for teachers to plan and implement.

Units comprise the selected topics of study for the grade level, organized into lessons. The units relate directly to the expectations identified on pages 6 through 12, which complement those established in the Ontario Curriculum for Mathematics (2005).

The introduction to each unit summarizes the general goals for the unit and provides background information for teachers. Each unit begins with a list of books for students that relate to the unit; a list of related websites (for all units combined) can also be found on pages 59 and 60.

Units are organized into lessons, based on the outcomes.

Note: This does not imply that a lesson can be covered in only one lesson period; many will carry over several lesson periods.

The lessons are arranged in the following format:

Background Information for Teachers:
Some lessons provide teachers with the basic mathematical knowledge they will need to present the activities. This information is offered in a clear, concise format, and focuses specifically on the topic of study.

Materials: A complete list of materials required to conduct the main activity or activities is provided. It includes classroom materials, equipment, and visuals. The quantity of materials required will depend on how you conduct activities and whether students are working individually or in groups.

Activity/Activities: This section details a step-by-step procedure, including higher-level questioning techniques and suggestions for encouraging active inquiry and discussion.

Activity Sheets: Reproducible activity sheets have been designed to correlate with the specific expectations of the activity or activities. Many of these are used during the activity to record results of investigations. Others are used as follow-up to the in-class activities. Students may work independently on these sheets, in small groups, or you may choose to read through them together and complete them in a large group setting. Activity sheets can also be made into overheads or large experience charts. Since it is also important for students to learn to construct their own charts and recording formats, these activity sheets can be used by the teacher as examples of ways to record and communicate ideas about an activity. Students can then create their own sheets rather than use the ones provided.

Note: Activity sheets are meant to be used only in conjunction with, or as a follow-up to, the hands-on activities. The activity sheets are not intended to be the mathematics lesson in itself or the sole assessment for the lesson.

Problem Solving: Many lessons include suggestions for problem-solving activities that are directly related to the lesson’s expectations. These problems may be presented orally, acted out with concrete objects, presented pictorially, or written out on chart paper. At the end of each unit, many of these problems are presented again on black line masters. Teachers can copy these sheets onto overhead transparencies to present to students as daily problem-solving activities. Or, the masters can be copied for students and cut apart, problem by problem. Students can then paste the problems into their math journals or agendas for completion independently.
Extension: Many lessons include optional activities to extend, enrich, and reinforce the expectations.

Activity Centre: Some lessons include independent student activities that focus on the expectations.

Assessment Suggestions: Throughout each unit, several suggestions are made for assessing student learning. Again, these assessment strategies focus specifically on the expectations of a particular activity topic. In the next section of the *Hands-On Mathematics* program, assessment is dealt with in detail. Keep in mind that the suggestions made within activities are merely ideas to consider; you may use your own assessment techniques or refer to the other assessment strategies on pages 18 and 19.

Classroom Environment

The classroom setting is an important component of the learning process. An active environment – one that gently hums with the purposeful conversations and activities of students – indicates that meaningful learning is taking place. While studying a specific topic, the room should display related objects and materials, student work, pictures and posters, maps, graphs, and charts made during activities, and summary charts of important concepts taught and learned. These reinforce concepts and skills that have been stressed during mathematics activities.

Timelines

No two groups of students will cover topics and material at the same rate. Planning the duration of units is the responsibility of the teacher. In some cases, the activities described will not be completed during one block of time and will have to be carried over. (Division of units into “lessons” does not imply that they can be covered in only one lesson period. Many lessons will, in fact, carry over several lesson periods.) In other cases, you may observe that the students are especially interested in one topic, and you may choose to expand upon it. The individual needs of your students should be considered as there are no strict timelines involved in the *Hands-On Mathematics* program. It is important, however, to spend time on every unit in the program so that students focus on all of the expectations established for their grade level.

Classroom Management

Although active learning is emphasized throughout this program, the manner in which these experiences are handled is up to you. In some cases, you may have all students working with materials and resources individually; in others, you may choose to use small group settings. The latter encourages the development of social skills and enables all students to be active in the learning process; it also means less cost in terms of materials and equipment. Again, classroom management is left up to you, since it is the teacher who ultimately determines how the students in his/her care function best in the learning environment.
Planning Guidelines

Mathematics is a skills-based subject. In order to acquire these skills, students need to visit and revisit them over the course of the school year.

*Hands-On Mathematics* is organized into strand- or topic-focused units. This organization allows teachers to follow the development of concepts from introduction to mastery within a given grade level. To ensure that students have opportunities to develop their mathematical skills in all topics throughout the year, it is recommended that teachers address concepts from each of these units in every reporting period or school term. For example, teachers may choose to begin the year with the unit on Patterns and Relations (Unit 1) but should continue to develop students’ skills in this area throughout the entire school year through review, continued practice, and new mathematical challenges. In the same way, although the unit on Number Operations (Unit 6) is presented last in the *Hands-On Mathematics* program, students should be provided with opportunities to review, practice, and investigate number operations throughout the school year.

Planning in this way gives students the time needed to solidify their understanding and, at the same time, helps to keep the concepts and vocabulary in the forefront throughout the year.

**Note:** Developing a year plan will ensure that topics are dealt with throughout the school year. Teachers can design the plan to meet their students’ specific needs and fit into their school calendars. On the following two pages, a sample year plan template is provided. Teachers can use the template to record the skills from each unit that they will teach during each month. There is also additional space for briefly noting other curriculum connections and themes that may relate to the overall math plan. Teachers can divide the bottom row of the chart according to the duration of the curriculum connection and related theme, which may be less than or greater than one month.

**Note to Teachers**

2 Sorting and Graphing Apples

Materials
- a variety of apples (You will need at least three varieties/colours of apples: one red, one green, and one gold apple. Have enough apples for each student to taste a piece of each variety.)
- knife
- paper plates
- pencils
- apple templates (included. Photocopy, and cut apart along dotted lines. You will need two paper apples per student.) (2.2.1)
- crayons, pencil crayons, or markers
- scissors
- chart paper
- markers
- overhead transparencies of Activity Sheet A (2.2.2) and Activity Sheet B (2.2.3)
- overhead projector
- non-permanent overhead markers
- two Hula-Hoops (or two long pieces of string, formed into loops)
- large sheet of Bristol board
- index cards
- round dot stickers

Activity: Part One: Sorting Apples
Display the collection of apples, and have students examine them. Ask:
- How are these apples the same?
- How are they different?
Discuss the colour and size of the various apples. Ask:
- Can you put the apples in order from smallest to largest? Lightest to heaviest?
As a class, put the apples in order by these criteria. Ask:
- How can we sort the apples?

Brainstorm sorting rules, and have students sort the apples in a variety of ways. Discuss sorting rules and attributes (colour, shape, stem/no stem, texture, and so on).

Activity: Part Two: Graphing Apples on a Pictograph
Safety Note: In this activity, students taste apples. Be aware of any student allergies before doing the activity.
Select three varieties/colours of apples to use for this graphing activity. Display the apples, and discuss their names and features. Explain to students that they will each taste the three varieties of apples and pick their favourite. Ask:
- Which apple do you think you will like the best?
Provide each student with a paper plate. Ask:
- How could we divide each plate into three equal parts?
Discuss students’ ideas, using the opportunity to talk about fractions. Then, have students draw pencil lines on their plates to divide them into three sections, one for each type of apple. Have students label the sections with the names of the three varieties of apples.

Now, cut up the apples so that there is one piece of each variety for each student. Leave one apple of each variety intact to serve as an example.

Safety Note: Be sure students wash their hands before and after handling the apples.

Distribute the apple pieces, and have students place them on the appropriately labelled sections on their paper plates. Then, have students sample each variety of apple and decide which kind they like best.
When students have decided on their favourites, give each student a blank apple template (2.2.1), and have students colour their apples the same colours as their favourite types of apples. Also, have students write their own names on their paper apples, and cut the apples out.

**Note:** Encourage students to colour their paper apples as accurately as possible, looking closely at the colours on the real apples and trying to replicate them. This may mean having two shades or colours on one paper apple, since many apples are not one solid colour.

Explain to students that they will now graph the results of this activity to show what type of apple each student in the class likes best. With student input, draw a large pictograph on chart paper. Have students place their coloured paper apples in the correct locations on the graph, as in the example below:

**Our Favourite Apples**

<table>
<thead>
<tr>
<th>Type of Apple</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Golden Delicious</td>
<td>3</td>
</tr>
<tr>
<td>Macintosh</td>
<td>4</td>
</tr>
<tr>
<td>Granny Smith</td>
<td>2</td>
</tr>
</tbody>
</table>

When the pictograph is complete, discuss the data by asking students:

- Which is the favourite variety of apple?
- How many more students like MacIntosh apples than Golden Delicious apples?

**Activity: Part Three: Sorting Apples on a Venn Diagram**

Place two Hula-Hoops or string circles on a large sheet of Bristol board to create an intersecting Venn diagram. Use two index cards to label the circles “Red” and “Green,” as in the following example:

Provide each student with another apple template and again have each student colour his/her paper apple the colours of the apple variety he/she likes best. Have students cut out their paper apples and print their own names on them.

Now, have students look carefully at the Venn diagram. Ask:

- If your apple is completely green, where on the Venn diagram should you put your apple? What if your apple is completely red?
- Where should you put an apple that is both red and green?
- Where should you put a yellow apple?

Have students put their paper apples in the correct location on the Venn diagram.

**Activity: Part Four: A Survey About Apples**

Explain to students that they will now complete surveys to find out the different ways students like to eat apples. Brainstorm with students...
for ideas of how apples can be prepared. For example: apple pie, apple crisp, apple sauce, caramel apple, baked apple, dried apple, apple chips.

Display the overhead copy of Activity Sheet A (2.2.2). Explain to students that they must first choose three “ways” of eating apples for their surveys and print these at the tops of their charts. Then, they will survey their classmates to find out which of those three ways of eating apples each student prefers.

Remind students that they need to record the names of students they question, to ensure that they survey the entire class. When they complete their surveys, students will also need to calculate the total number of names they have recorded in each column of their charts.

Allow students time to circulate and collect data. During this time, complete the survey process yourself, and record results on your transparency copy of Activity Sheet A (2.2.2), to use later for demonstration.

**Activity Sheet A**

**Directions to students:**

Choose three “ways” of eating apples, and record these on your chart. Survey your classmates to find out which of these ways of eating apples is each student’s favourite (2.2.2).

When students have finished collecting their information, explain that they will now transfer the information onto a bar graph. Display the overhead transparency of Activity Sheet B (2.2.3). Remind students that the bars on a bar graph should have spaces between them.

Discuss the data you collected on your copy of Activity Sheet A (2.2.2) by showing students your apple foods survey and asking:

- Which “way” of eating apples is the least favourite?

Distribute Activity Sheet B (2.2.3), and have students use the data they collected from their apple foods surveys (2.2.2) to complete their own bar graphs.

**Activity Sheet B**

**Directions to students:**

Use the data from your survey to construct a bar graph that shows your classmates’ favourite ways of eating apples (2.2.3).

**Activity: Part Five: Line Plots**

As a class, use line plots to graph the results from Activity: Part Two and Activity: Part Four. Create large line plots on graph paper. Use coloured dot stickers as plotting dots, as in the example below:

<table>
<thead>
<tr>
<th>Our Favourite Apples</th>
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</thead>
<tbody>
<tr>
<td>Golden Delicious</td>
</tr>
<tr>
<td>MacIntosh</td>
</tr>
<tr>
<td>Granny Smith</td>
</tr>
</tbody>
</table>

**Problem Solving**

An average person eats 3 apples per week. How many apples would an average person eat in a month? How many apples would 10 average people eat in a week? How many apples would a class of students the size of yours eat in a week?

**Note:** A reproducible master for this problem can be found on page 236.
Extensions

- Add the terms *Venn diagram*, *data*, and *information* to your classroom Math Word Wall.

- Plan a field trip to a grocery store. Visit the produce department to do further research on apple varieties. Prepare a list of questions to ask the produce manager.

- Collect apple recipes, and create a class cookbook. Test each recipe by making it in class.

Assessment Suggestion

Distribute math journal sheets (page 30), and have students record what they learned during the survey process and by preparing their bar graphs (Activity: Part Three).
Apple Templates
# Ways of Eating Apples Survey

<table>
<thead>
<tr>
<th>Name</th>
<th>Ways of Eating Apples</th>
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**Total**