hands-on problem solving

A Minds-On Approach

Grade 6

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Contents

Introduction to Hands-On Problem Solving Grade 6

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Program Introduction

Hands-On Problem Solving focuses on developing students' knowledge, skills, attitudes, and strategic thinking related to mathematics through active inquiry, problem solving, and decision making. Throughout all activities presented in the book, students are encouraged to explore, investigate, and ask questions in order to heighten their own curiosity about and understanding of the world of mathematics.

Program Principles

1. Effective problem-solving programs involve students actively building new knowledge from experience and prior knowledge.
2. Development of students' understanding of concepts, flexibility in thinking, reasoning, and problem-solving skills/strategies form the foundation of the problem-solving program.
3. From a young age, children are interested in mathematical ideas. This interest must be maintained, fostered, and enhanced through active learning.
4. Problem-solving activities must be worthwhile and relate to real-life experiences. Problems should be rooted in context so that students can make sense of the numbers with which they are being asked to work in a meaningful way.
5. The teacher's role in the problem-solving process is to actively engage students in tasks and experiences designed to deepen and connect their knowledge. Students learn best by doing, rather than by just listening. The teacher, therefore, should focus on creating opportunities for students to interact in order to propose mathematical ideas and conjectures, to evaluate their own thinking and that of others, and to develop mathematical problem-solving skills.
6. Problem solving should be taught in correlation with the mathematics program and with other school subjects. Themes and topics of study in problem solving should integrate ideas and skills from mathematics, as well as from other areas of study, whenever possible.
7. The problem-solving program should encompass and draw on a range of educational resources including literature and technology as well as people and places in the local community.
8. Assessment of student learning in problem solving should be designed to focus on performance and understanding and should be conducted through meaningful and varied assessment techniques carried on throughout the modules of study.

Big Ideas in Mathematics

In order to achieve the goals of mathematics education and to support lifelong learning in mathematics, students must be provided with opportunities to encounter and practise critical mathematical processes. Problem solving is one of these processes, but since they are all inter-related, it is important to recognize the characteristics of each mathematical process, and the related learning experiences for students. These processes are as follows:

Communication

Students need opportunities to share their mathematical ideas and thinking through oral language, reading and writing, diagrams, charts, tables, and illustrations. Communicating mathematically, aloud, or on paper, helps students clarify their thinking for themselves and others.

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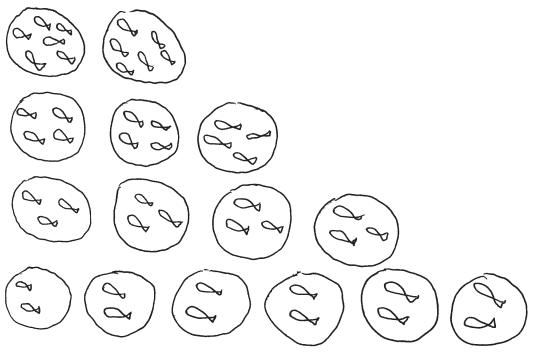
For example:

There are 12 goldfish.

The goldfish are in fishbowls.

Each bowl has the same number of goldfish in it.

Show different ways the goldfish could be put into fishbowls.



The process of communication is essential to the learning process during problem-solving investigations. Students should be encouraged to share their ideas, listen to others, and write about their problem-solving experiences, strategies, and solutions. In addition, students should be encouraged to write their own problems.

Connections

When doing problem-solving activities in the classroom, teachers should ensure that links are made between the various strands of the mathematics curriculum. It is also important to make connections between concrete, pictorial, and symbolic representations, so students should be encouraged to explore the use of manipulatives, illustrations, and symbols to solve problems. Further, concepts and skills should be connected to everyday life and to other curricular areas.

Mental Math

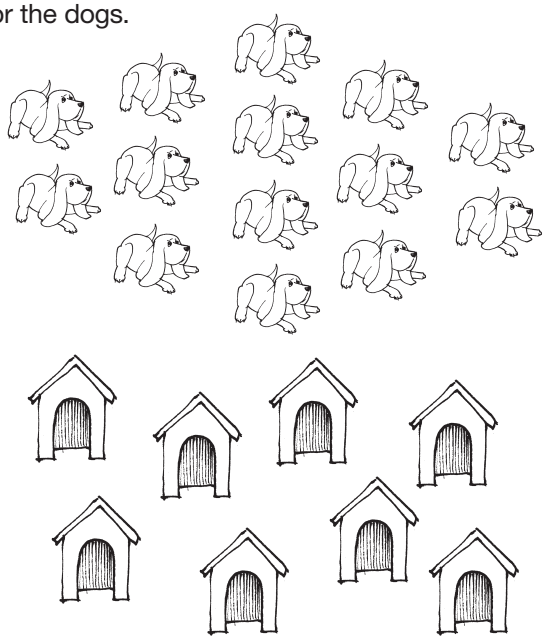
Mental math is more than just knowing the facts—it is about strategic thinking and number sense. Mental math is a process necessary to many everyday experiences, and students need extensive exposure to activities that encourage them to solve problems mentally. Strong mental math skills enable students to respond quickly to questions or required tasks phrased in a variety of ways. For example:

- Double 7
- Half of 12
- Six 9s
- How many legs on 8 spiders?

Estimation

Students should be encouraged regularly to estimate quantities and measurements. Being able to make an educated guess allows students to independently check the validity of their calculations. It is also an essential skill in everyday life. Estimation encourages students to take risks, use background knowledge, and learn from the process. For example:

Estimate whether there are enough dog houses for the dogs.



Now, check. Are there too many or too few dog houses?

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Reasoning

Mathematical reasoning involves informal thinking, conjecturing, and validating. Students should be encouraged to justify their solutions, thinking processes, and hypotheses. Good reasoning is as important as finding correct answers, so students need many opportunities to think about, describe orally, and record their mathematical activities and ideas. For example:

I am a 2-digit number.

My tens digit is 2 greater than 3.

My ones digit is 3 less than my tens digit.

What number am I?

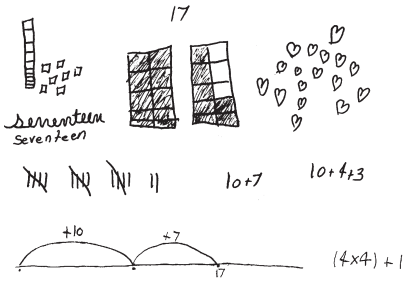
Technology

Use of calculators is recommended, to facilitate and enhance problem-solving skills and encourage discovery of number patterns. However, calculators must not replace development of students' number concepts and skills. Other technologies, like interactive whiteboards, computer software, and websites can also provide valuable resources for students and teachers.

Visualization

This is the process of creating mental images needed to develop concepts and understand procedures. Visualizations help students clarify their understanding of mathematical ideas. For example:

- Show all you know about the number 17. Use pictures, diagrams, and words in your answer.



Problem Solving

Problem solving is another of the "big ideas" in mathematics—the mathematical processes students need in order to achieve the goals of mathematics education and to support lifelong learning in mathematics. Students are exposed to a wide variety of problems in all areas of mathematics in ***Hands-On Problem Solving***. They also explore a variety of methods for solving and confirming their solutions to a variety of different types of problems. They should also be encouraged to find multiple solutions for problems and to create their own problems.

What Is Problem Solving?

Problem solving refers to "mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development" (Cai and Lester, NCTM). Problem solving is the application of a variety of mathematical tools, strategies, and knowledge to a wide range of math problems in order to solve them.

Problem solving

- Is a life skill;
- Creates a purpose for learning skills and concepts;
- Motivates students by developing a sense of inquiry;
- Allows students to demonstrate their understanding of mathematical concepts and skills in meaningful contexts;
- Teaches perseverance.

Problem solving should be the main focus of mathematics instruction. The ability to apply their knowledge to solve problems is the goal for all students.

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Best Practices in Teaching Problem Solving

Problem solving is often not viewed positively by students. In order to change this perception teachers should

- Use a problem-solving approach when introducing and teaching concepts and skills;
- Begin with simple problems so students can experience success;
- Include a balance of routine, non-routine, and extended exploration problems;
- Encourage the use of multiple strategies for solving problems;
- Provide opportunities for students to write their own problems;
- Use modelling (think aloud) to demonstrate the thinking processes involved in solving a problem. Students will be reluctant to attempt a problem if they do not know where or how to begin.
- Provide time for reflection (journaling, summarizing, and so on) in order to clarify mathematical ideas and relationships;
- Encourage discussion (turn-and-talk, whole class, and so on) to develop and reinforce critical and creative thinking skills.

Routine Problems

These are problems in which the way to a solution is generally straightforward. The solution usually involves one or two arithmetic operations.

Problem Types

Efforts are made to offer a variety of types of routine problems for students to solve in ***Hands-On Problem Solving***. As such, those problems focusing on number concepts include the following operations and problem types:

- Addition and subtraction: Beginning unknown, middle unknown, and end result unknown
- Multiplication: Product unknown
- Division: Quotitive and partitive division

These problem types are described in detail in the Implementation of Routine Problems section (see page 24).

Non-Routine Problems

These problems are more challenging for students. Upon first reading, the path to a solution is not immediately evident. Students draw on a bank of strategies (teacher-presented and student-developed) to solve the problem. Possible strategies include the following:

1. Act it out/use materials.
2. Draw a picture/diagram.
3. Look for a pattern.
4. Use logical reasoning.
5. Guess and check.
6. Make an organized list.
7. Make a table.
8. Work backwards.
9. Use an equation.
10. Use simpler numbers.

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Some non-routine problem-solving strategies are more appropriate for use at specific grades than others. The chart below provides details for when each strategy is addressed in the ***Hands-On Problem-Solving*** program:

Descriptions of these strategies are provided in detail in the Implementation of Non-Routine Problems section (see page 122).

Strategy	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Act it out/use materials	✓	✓	✓	✓	✓	✓	✓	✓
Draw a picture/ diagram	✓	✓	✓	✓	✓	✓	✓	✓
Look for a pattern	✓	✓	✓	✓	✓	✓	✓	✓
Use logical reasoning	✓	✓	✓	✓	✓	✓	✓	✓
Guess and check		✓	✓	✓	✓	✓	✓	✓
Make an organized list		✓	✓	✓	✓	✓	✓	✓
Make a table			✓	✓	✓	✓	✓	✓
Work backwards					✓	✓	✓	✓
Use an equation					✓	✓	✓	✓
Use simpler numbers					✓	✓	✓	✓

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Extended Exploration Problems

Extended exploration problems are meant to provide a thought-provoking challenge for students. These problems may present mathematical situations that are slightly beyond the grade-level curricular outcomes, may take the form of an investigation, or may require an extended period of time to solve. In all cases, students are encouraged to use their own strategies to arrive at a solution(s).

Extended problems are open ended, can be investigative in nature, and have multiple entry points to allow for differentiation. They often

- Have more than one solution/answer;
- Can be solved using a variety of strategies;
- Require students to gather their own data;
- Require creative and critical thinking;
- Require more/extended time to solve;
- Make connections to the real world.

Extended exploration problems support the other six "big idea" mathematical processes: communication, connections, mental math, estimation, reasoning, technology, and visualization. The engaging nature of these problems helps students develop perseverance.

Examples and procedures for extended explorations are described in detail in the Implementation of Extended Exploration Problems section (see page 216).

Implementing the Hands-On Problem-Solving Program

Hands-On Problem Solving is arranged in a format that makes it easy for teachers to plan and implement, with tasks that relate to specific outcomes/learning expectations established in Canadian curriculum documents.

Program Format

Problem-solving tasks are presented as daily mathematics activities and are organized according to the approximate number of weeks in the school year. As such, there are 40 weeks-worth of problem-solving tasks, consisting of

- 40 routine problems that focus on math topics including number, patterns, measurement, and geometry. These problems are identified as problems 1A through 40A;
- 40 non-routine problems that focus on specific strategies for the grade level. These problems are identified as problems 1B through 40B;
- 10 extended exploration problems that offer in depth, real-life contexts as the basis for problem solving. These problems are identified as problems 1C through 10C.

Planning Your Year of Problem Solving

The three types of problems (routine, non-routine, and extended explorations) are presented in three separate sections of this book, each with its own detailed introduction on implementation. However, it is essential that students focus on all three types of problems throughout the school year. Therefore, it is recommended that teachers do one routine and one non-routine routine problem with students each week, and one extended exploration each month.

In the following section of ***Hands-On Problem Solving*** a correlation chart identifies the math concepts presented in each lesson of the book. Teachers can refer to this chart to plan problem-solving activities that correspond with other math activities occurring in the classroom. For example, if students are focusing on geometry in math, the correlation chart will show which problems herein connect to that topic.

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Curricular Connections

Efforts have been made to correlate ***Hands-On Problem Solving*** problems with other curricular areas, such as language arts, science, and social studies. For example, some problems connect specifically to a science or social studies topic or to a general area of emphasis such as social justice. As teachers become familiar with the problems, they will find opportunities to connect these problems to specific units or topics of study.

Supporting Literacy During Problem Solving

It is important that all students, regardless of reading ability, have the opportunity to participate and succeed in problem solving. As such, some will require additional supports to read and understand the problems presented. To help support students' literacy skills, consider the following options:

- Read the problem aloud, and have students follow along.
- Read the problem as a class.
- Have students work with partners or in small groups to read and discuss the problem.
- Introduce, discuss, and review related math vocabulary, and display pictorial representations in the classroom (for example, display labelled illustrations of prisms, pyramids, and cubes during a lesson in which students must draw on their knowledge of 3-D solids).

The Questioning Process

During the problem-solving process, it is important for teachers and students to pose questions and to consider various strategies for solving the problem. To encourage these processes, blackline masters of guiding questions have been included for teacher and student use (see page 10). These two templates (one for teacher use and the other for student use) provide suggested questions that can be asked during the problem-solving process.

The blackline masters can be photocopied onto sturdy tag board and laminated for long-term use. Teachers may choose to use these resources during lessons, as they support students in their problem solving. Students can glue their cards into problem-solving file folders or notebooks, or the cards can be placed on desks or tables for use during problem-solving activities.

Additional Resources

For some problem-solving tasks, students might use strategies requiring specific materials, such as hundred charts, number lines, graph paper, dot paper, and so on. These materials can be found in the Appendix at the back of the ***Hands-On Problem Solving*** book (see page 253); teachers are encouraged to photocopy these resources and distribute them to students as needed.

A Note About Pennies

The Government of Canada, in its 2012 Budget, announced its intention to withdraw the Canadian penny from circulation; as of February 4, 2013 the Royal Canadian Mint will no longer distribute pennies. However, the Government of Canada has also indicated that

- The cent will remain Canada's smallest unit for pricing of goods and services.
- The penny will retain its value indefinitely, and consumers can continue to use it in payments for goods and services.

Government of Canada Budget 2012 – Phasing Out the Penny <www.budget.gc.ca/2012/themes/theme2-eng.html> and Frequently Asked Questions: Consumers <www.budget.gc.ca/2012/themes/theme2-fs-fi-1-eng.html>

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Pennies are included in some problems in the ***Hands-On Problem Solving*** program. The rationale for this is that using pennies in a problem-solving context

- Supports counting skills;
- Builds familiarity with money;
- Lends itself to grouping and place-value structure of base ten;
- Prepares students for global citizenship. Many monetary systems still include a penny or other coin with a value of 1;
- Can extend to opportunities to explore other Canadian coins that are in circulation but may not be used on a regular basis (for example the 50 cent coin).

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Routine Problems

1A Number Stumper

Math Topic

Number

Math Concepts

- Place value
- Multiplication
- Fractions

Problem

Mr. Fodor's class is playing a math game called Number Stumper.

Josh gives his classmates some clues to help them guess his number:

- There is a 1 in the number.
- The digit in the hundreds place is 3 times the digit in the thousands place.
- The digit in the ones place is 4 times the digit in the tens place.
- The digit in the millions place is 1/2 the digit in the hundreds place.
- The digit in the hundred-thousands place is 1 less than the digit in the ones place.
- There is a 9 in the ten-thousands place.

What is Josh's number?

Background Information for Teachers

To support students as they approach this place-value problem, encourage them to create a chart to represent the place-value system (including labelled place-value names and numerals). This will help them to set up the problem and keep things organized.

Encourage students to do one step (clue) at a time and to put a checkmark in the left margin next to each clue, once they have addressed it.

Students may use a variety of strategies to solve this problem, such as the following:

- Draw a diagram of base-ten materials
- Use a place-value chart (see the solve section below for an example. Blank Place Value charts are included in the Appendix on page 254)

Think 

Provide time for students to read, think, and formulate ideas about the problem.

Talk 

Discuss the problem with students. Ask:

- What do you need to find out? (Josh's number)
- What information is important in the problem? (Highlight the important information.)
- What information is not important? (Underline the unimportant information.)
- Can you name the answer? (seven-digit number)
- In what ways can the problem be solved?
- What strategies can you use to help you solve the problem?

Solve 

Josh's number is 3 392 614.

Millions	3
Hundred Thousands	3
Ten Thousands	9
Thousands	2
Hundreds	6
Tens	1
Ones	4

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1A

Share 

Have students share their strategies and solutions.

Extend

Have students solve the following extension problem:

Create clues for your own number stumper.

Challenge a classmate to solve your problem.

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Date: _____ **Name:** _____

Number Stumper

Mr. Fodor's class is playing a math game called Number Stumper.

Josh gives his classmates some clues to help them guess his number:

- There is a 1 in the number.
- The digit in the hundreds place is 3 times the digit in the thousands place.
- The digit in the ones place is 4 times the digit in the tens place.
- The digit in the millions place is 1/2 the digit in the hundreds place.
- The digit in the hundred-thousands place is 1 less than the digit in the ones place.
- There is a 9 in the ten-thousands place.

What is Josh's number?

Think

Talk

Solve

Share

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